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Experiments involving Random Matrices within the context of Numerical Analysis

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Abstract
We present a set of experiments concerning different aspects of the analysis of random generated data (stored in matrix form). Our goal is to display some pros and cons of the use of such type of data, under the context of Numerical Analysis.

Key words:
Numerical Analysis, Random Matrices, Numerical Experiments.

Introduction

We divided our investigation into four distinct classes of experiments:

• Experimental analysis of eigenvalues and singular values of some classic Random Matrix ensembles (e.g. Gaussian Orthogonal Ensemble). In this class of experiments, we present some conjectures about what we have found out. Our starting point was [1]

• Theoretical and experimental analysis of eigenvalues of Haar-distributed Orthogonal Matrices. We aimed to prove the convergence of the empirical distribution of the eigenvalues, as the dimension grows, to the circular uniform distribution.

• Qualitative analysis of a matricial 1-norm estimation algorithm, under random generated data, mostly following the same steps described in [2].

• Theoretical and experimental analysis of the eigenvalues of another specific ensemble of random generated matrices. Our intention here is to present the convergence (again, as the matrix dimension tends to infinity) of the empirical distribution of these eigenvalues to the Marchenko-Pastur distribution [3].

Results and Discussion

As we progressed into the experimental journey, we could certainly signal some quality traits and disadvantages on the use of random generated matrices, for the numerical experiments we performed, and, within a certain degree, infer these points about other experiments (still in reach of a numerical analyst). In the following images, we show the visual results of the experimental analysis referent to the second class, described above.

Image 1. Set of 400 uniformly distributed points in S^1.

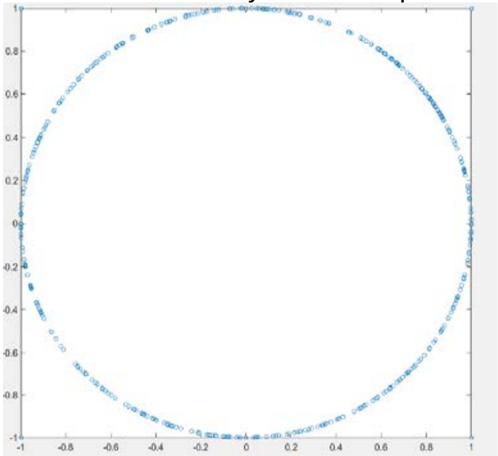
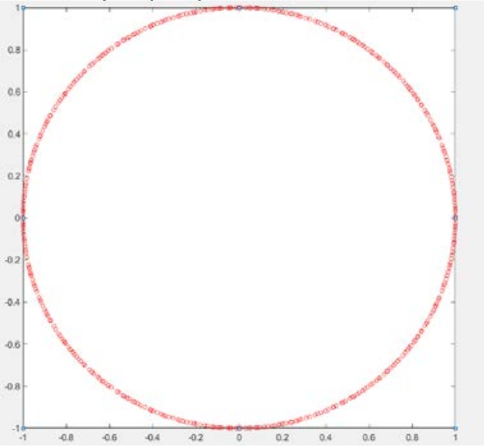


Image 1. Eigenvalues of a Haar-distributed matrix in the Orthogonal Group O(400).



Conclusions

Over the course of the experiments, we could infer certain conclusions about the aspects of the use of random generated matrices, in the numerical context. They all revolve around the fact that random matrices do incorporate information according to their structure; a numerical analyst that embraces this fact could use such a behavior to somehow improve his numerical algorithms; however, one that does not, could have their experimental inferences obstructed by the influences generated by these intrinsic features.

In general, we conclude that a numerical analyst should be aware of this inherent features of random matrices.

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