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Studying the existence of limit cycles of piecewise differential equations using the Newton-Kantorovich method

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The study of differential systems has always been of interest, as its applications are rich – they model, for example, biological phenomena, chemical reactions, and the motion of galaxies [5]. When studying the periodic behaviors of these systems, one of the main questions is the analysis of the periodic orbits and, in particular, the existence and number of isolated periodic orbits, which are called limit cycles. Ilyashenko showed that a system only has a finite number of limit cycles, however, the maximum number of limit cycles of a system is not yet known for generic systems of order $n > 1$ [2].

Starting with Andronov, a new class of differential systems began to be studied: the discontinuous or piecewise differential systems (PWDS). The analysis of these systems has been fast and of great interest in the last decades, as they model cell activity, control theory mechanisms, mechanical devices, and other phenomena of engineering, social science, pure and applied math, and others.

The study of PWDS is rich even in low dimensions. For example, while linear continuous differential systems do not have limit cycles, there are planar PWDS with at least three limit cycles [4].

In this work we investigated limit cycles of linear systems in $\mathbb{R}^3$ with three zones. In particular, we develop a numerical method to study the existence of periodic orbits of the system

$$\dot{\mathbf{x}} = \begin{cases} A_1\mathbf{x}, & \text{if } y < 0, \\ A_2\mathbf{x}, & \text{if } 0 < y < 1, \\ A_3\mathbf{x}, & \text{if } y > 1, \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^3$ and $A_i \in M_{3\times 3}(\mathbb{R})$ are matrices with three real eigenvalues.

In particular, we obtain a system with only one periodic orbit through the three zones. Using the Nashed-Chen Theorem, we prove rigorously that this orbit is closed and isolated and, therefore, a limit cycle.

Theorem 1. *The system* (1) *with*

$$A_1 = \begin{bmatrix} 1 & 0.15 & -0.5 \\ 0.15 & 1 & 0.5 \\ 1 & 1 & 0.15 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.195 & -0.42 & 0.5 \\ -0.42 & 0.195 & 0.2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.233961 & 0.418193 & 0.492521 \\ 0.418193 & -0.233961 & -0.492521 \\ 1 & 1 & 0.418193 \end{bmatrix}$$

has a limit cycle.

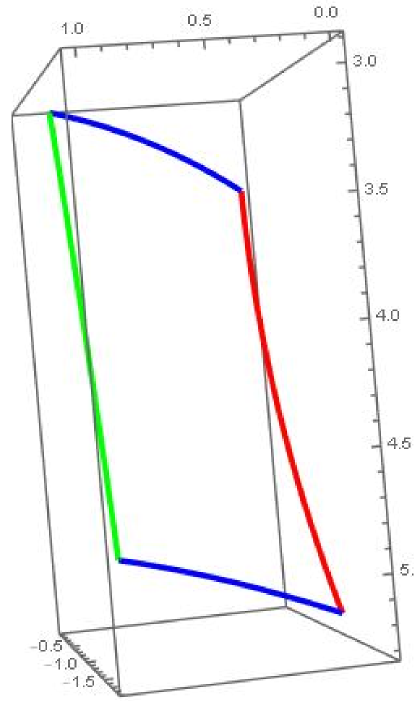

Figure 1: Limit cycle of Theorem 1.

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