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Description and Characterization of Optical Cavities Based on Photonic Crystal

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The project had the goal to study the electromagnetic wave propagation in photonic crystal (PhC), which is a system with modulated periodicity in the dielectric constant. As the electronic case, the PhC presents a photonic band diagram that we can determine the light propagation inside the crystal. Due to the existence of photonic band gap also there is the prohibition of wave propagation of certain frequencies. Inserting defects in the periodicity, it's possible to create waveguides and optical cavities. A theoretical study was made to understand the PhC's accompanied by numerical simulations using Comsol® and by experimental investigation. To this, it was measured optical transmission to characterize single and coupled cavities in Silicon PhC.

INTRODUCTION

Currently, photonics [1] is arising as an alternative method of information transport. In this way, it is interesting the storage and the light transport in microscopic devices through optical cavities. These cavities confine light in a spatial region contemning material or not, one example is the photonic crystal [2–4]. These systems have a periodic dielectric constant and a photonic band diagram which represents the light dispersion in the material. In the PhC, the cavity is created by an interference process in the unit cells of propagating waves. The vantages are nanocavities with a high-quality factor [5, 6] that can be coupled [7] and can be made of Silicon, therefore produced on a large scale [8–10]. Specifically, in this project we worked with a PhC proposed by Noda et al. [5], figure 1.

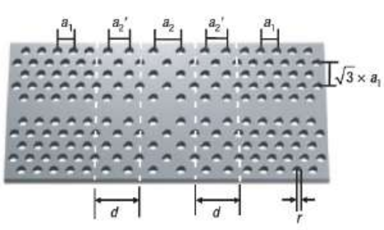

FIG. 1. PhC is made of a Silicon slab with periodical air holes responsible for the dielectric constant difference. There is a waveguide due to a linear defect in the middle and there is a cavity due to a lattice parameter variance in the center. The values are a_1=410nm, a_2=420nm, a_2'=$(a_1+a_2)/2$, d around $2a_2'$ and r around $0.26a_2$. Picture adapted of [5].

According to our progress in this work, we'll understand why these defects create a cavity. At last, we'll see how to work the coupling between two cavities, i.e., when one cavity electromagnetically interferes with the other.

PHOTONIC CRYSTAL

One-Dimensional photonic crystal

Working with Maxwell's equations in a material without current or free charges, it is obtained an eigenvalue equation to the magnetic field (1), called Master Equation [11] (it is possible to find the electric field in the same way).

$$\vec{\nabla}\times\left(\frac{1}{\epsilon(\vec{r})}(\vec{\nabla}\times\vec{H})\right)=\left(\frac{\omega}{c}\right)^2\vec{H}(\vec{r}). \tag{1}$$

To understand the equation we use some concepts from solid-state. First, the periodic dielectric constant $\epsilon(\vec{r})$ is localized by a lattice vector $\vec{R}$, which is a linear combination of primitive vectors (minimal vectors that allow describing the whole crystal). Besides that, we can construct a parallel space to the real space through a Fourier Transform of the magnetic field function. This step results in a wave vector space described by the reciprocal vector $\vec{G}$, with the condition $\vec{R}\cdot\vec{G} = 2\pi\delta_{ij}$. To solve the equation (1), we apply the Bloch Theorem that allows the expansion of the magnetic field in a linear combination of plane waves. In a one-dimensional photonic crystal, i.e., a structure with periodicity in one direction and homogeneous in the others, writing the magnetic field in terms of plane waves with the coefficients as a periodic function is obtained a dispersion equation to the light in the PhC given by $\frac{\omega}{c} = \frac{|\vec{k}|}{\sqrt{\epsilon}}$. According to this equation, the light frequency depends on the wave vector and the dielectric constant. So, if the structure has is just one dielectric constant, the band diagram will be continuous. However, if the structure has two dielectric constants, it will arise a discontinuity in the edges (where the dielectric change equals $\Delta\epsilon$). Therefore will be two frequencies for the same $\vec{k}$ to satisfy the dispersion equation creating a photonic band gap. One example of a one-dimensional photonic crystal is a multilayer film, where the band diagram is shown in figure 2 for different materials. It's important to note that the band repeats each $\vec{G}$, consequently, all band diagrams are equal n times of the first band, called First Brillouin Zone (BZ). In more dimensions, this zone is reduced by translational and rotational symmetries, forming the Irreducible Brillouin Zone (IBZ).

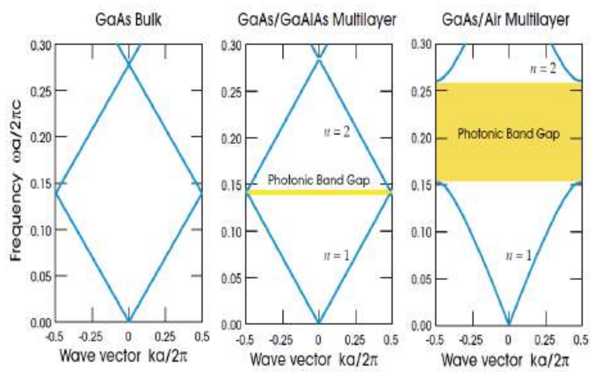

FIG. 2. Left: Band diagram of a system contemning just GaAs. Center: Band diagram of a multilayer film of GaAs/GaAlAs, as the $\Delta\epsilon$ is small, the discontinuity is small. Right: Band diagram of a multilayer film of GaAs/Air, here the $\Delta\epsilon$ is bigger, consequently the gap is bigger too. The normalized graphs are frequency per $\vec{G}$. The figure was taken from [11].

Two-Dimensional Photonic Crystal

Now, we start to understand the Noda crystal doing a simple two-dimensional PhC unit cell. All studies did here were made in the Comsol® , a finite element method software. The structure has periodicity in two directions, meanwhile is finite in the third direction. The hexagonal unit cell is shown in figure 3(a).

As the one-dimensional PhC, there is a band diagram, but in more dimension, the primitives vector form a polygon in the reciprocal space. Using arguments of translational and rotational symmetries it is possible to describe the light behavior in the crystal surrounding the IBZ. In the hexagonal case, spatial and reciprocal vectors are $\vec{R}_n = n_1(a,0) + n_2\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$ and $\vec{G}_m = m_1\left(\frac{2\pi}{a}, \frac{-2\pi}{\sqrt{3}a}\right) + m_2\left(0, \frac{4\pi}{\sqrt{3}a}\right)$. The BZ points is $\Gamma = (0,0)$, $\quad M = \left(\frac{\pi}{a}, 0\right)$ and $K = \left(\frac{\pi}{a}, \frac{\pi}{\sqrt{3}a}\right)$.

The band diagram of the cell is shown in figure 3(b). Simulating half of the geometry in $\hat{z}$ is possible performing Perfect Electric Conductor (PEC) or Perfect Magnetic Conductor (PMC) on the surface, separating the even and odd modes in the same direction. Even modes maintain the signal of the fields in the two halves of geometry, and the signals are inverted in odd modes. We note that just in z-odd modes there is a photonic band gap to low frequencies, so we will use always these modes in $\hat{z}$. The light line is a limiter that can be interpreted as a limit to modes coupling in the crystal, outside the light line the modes are in the air box. Some examples of modes are represented in 3(c). To z-even modes are used the E_z component because E_x and E_y in the middle of the cell is negligible, already to z-odd modes are used the E_y component because this is maximum meanwhile E_z is negligible.

Following the idea to construct a Noda crystal, we simulated a waveguide made of the same unit cell used before, figure 4(a). The waveguide appears due to a linear defect in the middle. As the defect is minimum in the structure, doesn't cause a change in the band diagram but allows guided modes inside the photonic band gap. These modes are trapped because of their evanescent aspect. The band diagram is shown in figure 4(b), it is noted that the frequency modes are approximately inside the band gap of the hexagonal unit cell diagram. After that, we changed the lattice parameter value fixing $k_x = \pi/a$, figure 4(b), the mode frequencies decrease according to the lattice increase. Using this fact, decreasing the lattice parameter slowly it's possible to trap the mode in a small model volume as a perturbation, smaller than a linear defect, creating a cavity.

The cavity geometry is similar to figure 1, the values are a_1=410nm, $a_2' = 420$ nm, $a_2 = 430$ nm and $r = 117$ nm. Besides that, it was found high order mode, this mode have more electric field nodes and lower quality factor, i.e, it's less trapped allowing radiation losses. One example is shown in figure 6(b), we simulated 1/4 of the simple cavity crystal using PMC on the boundaries surfaces and we found the even mode (maximum value to E_y component).

OPTICAL CAVITY

The optical cavities allow the electromagnetic field density storage in the device. One example is the Fabry-Perot cavity, figure 5.

If the fields in Maxwell's equations are expanded in plane wave base, using the orthogonality relations and

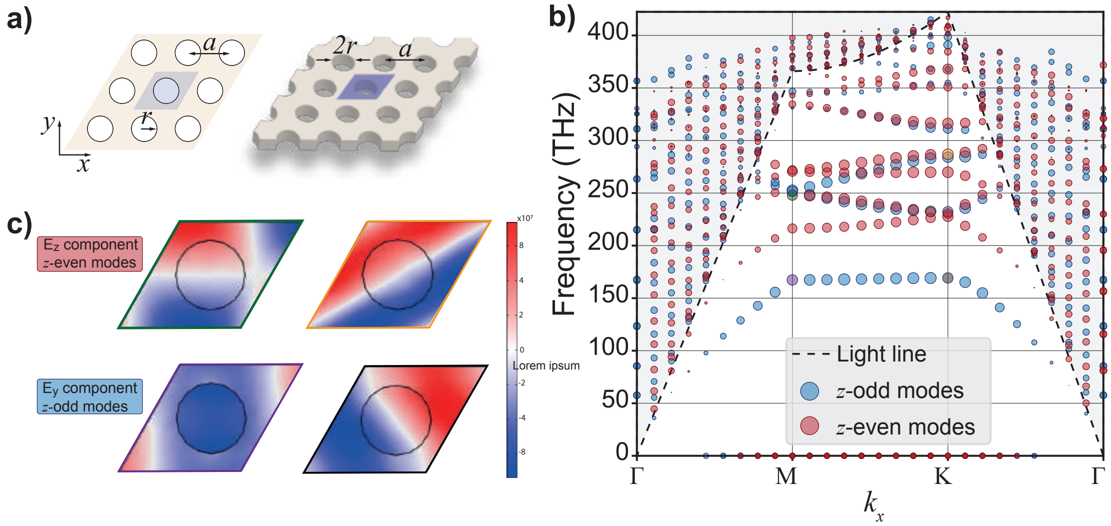

FIG. 3. (a) A hexagonal unit cell (highlighted in blue) with periodicity in the xy plane. The parameters are the lattice vector with $a = 410\,\text{nm}$, the radius with $r = 117\,\text{nm}$ and the thickness with $t = 250\,\text{nm}$. Also, there is an air block above the cell of $3\,\mu\text{m}$.(b) Band diagram of the hexagonal unit cell. The size markers are proportional to the quality factor and the electric energy density in the cell. The definition of quality factor is the mode frequency per loss, this concept will be explored more later.(c) Top line: Two z-even modes are shown, the color edge of the cell is used to represent the mode inside the band diagram, in M and K point respectively. Bottom line: Two z-odd modes are shown, the color edge of the cell is used to represent the mode inside the band diagram, in M and K point respectively.

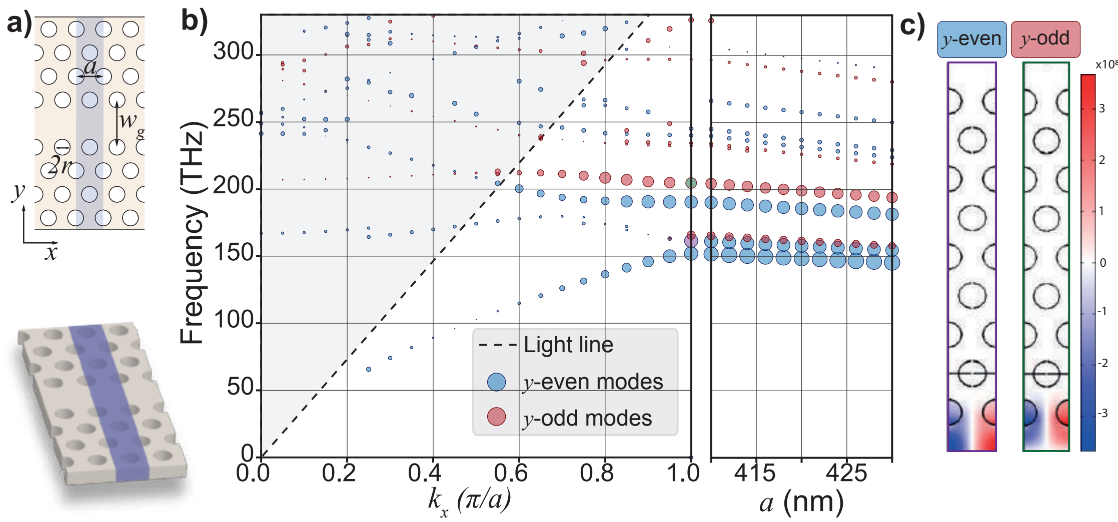

FIG. 4. (a) A waveguide with periodicity in the x direction. The BZ are highlighted in blue. (b) Left: Photonic band diagram of the waveguide. Right: Fixing the wave vector in π/a, the graph shows the mode behavior according to the lattice parameter value change.(c) Two waveguide modes are shown, one y-even and one y-odd, is represented E_y component and the color edge of the waveguide is used to represent the mode inside the band diagram.

expanding the fields also to optical modes, i.e., temporal and spatial modes $a_i(t)\varepsilon_i(\vec{r})$, we obtain the temporal evolution of optical mode. Defining a variable detuning $\Delta = \omega_l - \omega_c$ and changing the referential of the system to the laser frequency, we have:

$$\dot{a}(t) = i\Delta a(t) - \frac{\kappa_e + \kappa_i}{2}a(t) + \sqrt{\kappa_e}\alpha_{in}. \qquad (2)$$

where a(t) is the temporal mode, $\kappa = \kappa_e + \kappa_i$ is the total loss given by $\frac{\kappa}{2} = \omega_c \frac{\epsilon_i}{\epsilon}$ and α_{in} is the optical input. There are two losses types, one is the external arose of light coupling with the crystal and the other is an internal arose of intrinsic losses like scattering and absorption. Also, we put a phenomenological term associate with a loss of cavity with a laser excitation wave of frequency ω_l.

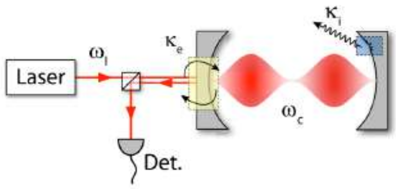

FIG. 5. Fabry-Perot cavity with a frequency mode of ω_c. The cavity is excited by an external frequency ω_l. The indicated losses are the external loss κ_e and the internal loss κ_i.

Using the input-output relations, $\alpha_{out} - \alpha_{in} = -\sqrt{\kappa_e} a$, in the stationary regime ($\dot{a}(t) = 0$), the transmission signal is the square of reflection amplitude module:

$$|R|^2 = |\frac{\alpha_{out}}{\alpha_{in}}|^2 = \frac{(\kappa_e - \kappa_i)^2 + 4\Delta^2}{(\kappa_e + \kappa_i)^2 + 4\Delta^2} = \frac{(2\kappa_e - \kappa)^2 + 4\Delta^2}{(\kappa)^2 + 4\Delta^2}. \quad (3)$$

the equation has the Lorentzian form.

To study modal coupling, we consider two nearby modes with weak coupling, the evolution of temporal mode is written as:

$$\frac{da_1}{dt} = i\omega_1 a_1 + i\kappa_{12} a_2 \quad and \quad \frac{da_2}{dt} = i\omega_2 a_2 + i\kappa_{21} a_1. \quad (4)$$

where κ_{ij} is the coupling rate of mode j in mode i. By energy conservation, we obtain that $\kappa_{12} = \kappa_{21}^* = \kappa$. If we expand the mode as $a_i \propto c_i e^{i\omega t}$, the coupling frequency is given by:

$$\omega = \frac{\omega_1 + \omega_2}{2} \pm \sqrt{\left(\frac{\omega_1 - \omega_2}{2}\right)^2 + |\kappa|^2} = \frac{\omega_1 + \omega_2}{2} \pm \Omega_0. \quad (5)$$

The coupling splits the frequency in two modes with difference equal $2\Omega_0$, a symmetric and an anti-symmetric mode. One example is shown in figure 6(d), we simulated 1/4 of the coupled cavity crystal using PMC on the boundaries surfaces and we found the even mode (maximum value to E_y component). We made the coupled crystal changing the lattice parameter value twice in the x direction, creating two perturbation regions.

FABRICATION AND OPTICAL CHARACTERIZATION

The microfabrication process used was corrosion by hydrofluoric acid. The steps are described in the following: in a Si wafer, there is a superior layer with the devices, where the photonic crystals are made of Si and the other regions of SiO_2. Over the wafer is deposited an ultraviolet sensitive resist, which is hydrofluoric acid resist. Using photon lithography is sensitized the interest region and made a revelation that takes off the resist of this region and maintains the other regions. After this step, the sample is put in the acid that corrodes just the SiO_2 in the layer with the devices and the PhC stays intact. The figure 6 show the two sample measured in the project, in figure 6(a) is the single cavity PhC and in figure 6(c) is the coupled cavity PhC.

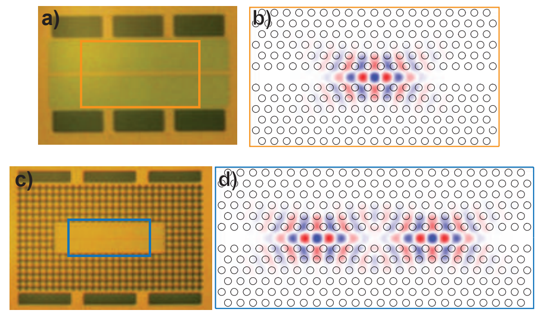

FIG. 6. (a) Photonic crystal with single cavity.(b) Optical mode simulated to single cavity.(c) Photonic crystal with coupling cavities, around the crystal there is a mechanical shield that supports the mechanical modes, but does not interfere in the optical mode.(d) Optical mode simulated to coupled cavity.

To characterization was fabricated tapers that are responsible by the light coupling in the cavity putting over the modal volume. The tapers are formed by a process of pre-derection and traction of conventional optical fibers. The experimental setup is illustrated in figure 7(a). The tuned laser is used to sweep the wavelength meanwhile the transmission of the cavity is collect by DAQ.The optical evanescent coupling between the taper and the optical cavity allows that the transmission spectrum be measured. As not it is possible read the wavelength direct from laser, we use MZ and HCN as assistant to calibrate the wavelength correctly.

The equation (3) was deduced to one port coupling, in the case of PhC we have two port coupling between the taper and the cavity. The new equation is:

$$|R|^2 = \frac{(\kappa - \kappa_e)^2 + 4\Delta^2}{(\kappa)^2 + 4\Delta^2}. \quad (6)$$

A important parameter is the quality factor which indicates the light storage without dissipation in the cavity. The definition of total and intrinsic quality factor are $Q = \omega_c/\kappa$ and $Q = \omega_c/\kappa_i$ respectively. In the transmission spectrum ω_c is the minimum value of resonance and κ is the half-height of Lorentzian. The measured spectrum of the crystal 6(a) is shown in figure **??**(b), the total loss is $\kappa = (923 \pm 1)$ MHz, the external loss is $\kappa_e = (76 \pm 1)$ MHz, the total quality factor is $Q = (211 \pm 1)$ thousand and the intrinsic quality factor is $Q_i = (231 \pm 1)$ thousand. The values are the according to literature [8]. To

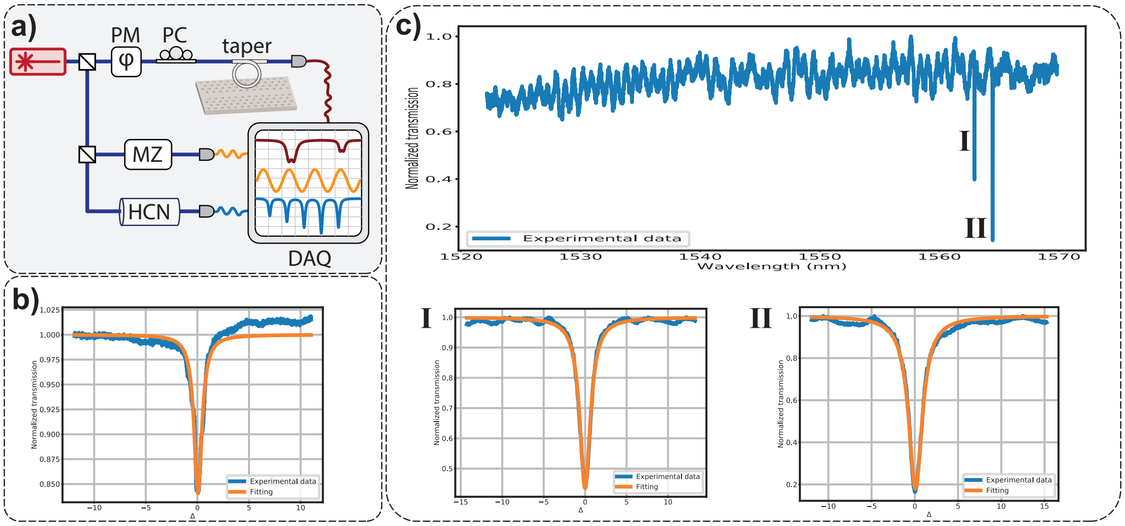

FIG. 7. **a)** The experimental setup is a tuned laser coupled in the cavity by the taper. The low frequency photodetectors (PD) collect the transmitted light that bring the optical information ((**b**)) and ((**c**)) from the cavity. MZ: Mach-Zehnder interferometer, MOD: Phase modulator, DAQ: digital analog converter, HCN: Hydrogen Cyanide.

the measurements of the crystal of the figure 6(b) we didn't find the split in the optical mode. A possible explanation is that the modes may be shifted from each other in the spectrum due to inaccuracy in the center frequency, the difference between the modal frequencies is approximately 1.96 nm which is equivalent to an inaccuracy of 0.13% in the center frequency or the modes do not coupled. In figure 7(c) are showed the two modes: to 7(c)(I) the total loss is $\kappa = (1.5 \pm 0.1)10^9$ Hz, the external loss $\kappa_e = (532 \pm 0.2)10^9$ Hz, the total quality factor is $Q = (122.8 \pm 0.1)$ thousand and the intrinsic quality factor is $Q = (186.2 \pm 0.1)$ thousand; to 7(c)(II) the total loss is $\kappa = (1.9 \pm 0.1)10^9$ Hz, the external loss $\kappa_e = (1.1 \pm 0.1)10^9$ Hz, the total quality factor is $Q = (99.9 \pm 0.1)$ thousand and the intrinsic quality factor is $Q = (236.2 \pm 0.1)$ thousand

CONCLUSION

In this work was studied system with modulated periodicity in the dielectric constant called photonic crystal. From solid state analogy, we studied the PhC constitution and the light propagation inside, creating waveguides and cavities. Also, we introduced the coupling cavities concept. All steps were accomplish with simulation in Comsol® . Experimentally it was measured optical transmission of single and coupled cavities in PhC, obtaining high quality factors. Unfortunately, not was possible to observe the split in the coupled cavities.

ACKNOWLEDGMENTS

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