INTRODUCTION TO GOAL PROGRAMMING APPLIED TO INVESTMENTS

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INTRODUCTION

Modern finance has expanded due to the use of increasingly sophisticated mathematical models, that help enlighten precise relations between phenomena that were previously unrelated or deemed independent. Accordingly, mathematical results in finance may provide valuable insights that would have been very difficult to discover otherwise and that have a deep economical signification (De Scheemaekere 2009).

Indeed, finance economics is a highly quantitative discipline, where mathematical complexity is comparable to that of natural sciences. However, even if financial models can be as complicated as those belonging to physics, they must not be evaluated according to the standards of natural sciences. Unlike the physicist working with constants and forces that are universal (energy conservation, gravitation, nuclear forces, etc.), the financial theorist deals with problems that are quite similar to the statistician's problems, facing a double uncertainty: the probabilistic nature of the outcome and the adequateness of the underlying hypotheses (De Scheemaekere 2009).

One inevitable issue to be addressed when selecting a portfolio is multiple criteria, because assets with a high return usually have high risk, implying that an optimal tradeoff must be found. Among the criteria imposed in this decision-making process are: liquidity, budget, cash income and industry/sector/issuer concentration constraints (Kim et al. 2022).

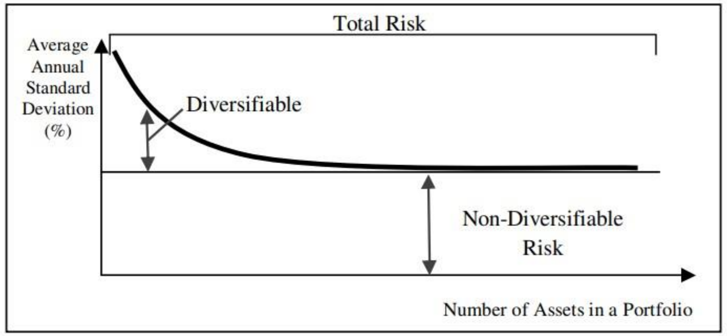

Fig 1: Diversifiable and Non-Diversifiable Risks (Azmi 2013).

Among the various methods for solving multi-objective optimization problems, there is Goal Programming (GP). In order to find a compromise solution among conflicting objectives, a set of acceptable goals is established a priori for each objective function (Arenales et al. 2007).

In this context, GP is a widely-used approach in the field of multiple criteria decision-making that incorporates variations of constraints and goals. To the original portfolio selection problem, with risk and return optimization, objectives representing other factors can be added for a more realistic approach to portfolio selection problems. The notion of programming in GP is associated with the development of solutions, or programs, for specific problems, and thus it has nothing intrinsically to do with computer programming. The name GP indicates seeking the optimal program for a mathematical model that is composed solely of goals (Azmi 2013).

In other words, GP is an optimization model that sets multiple targets and finds an optimal solution that closely reaches those target values. The overall objective is to minimize deviations between desired goals and actual achievements, while being able to emphasize more important goals. Besides addressing multi-objective characteristics in portfolio management, it is applied to areas such as supply chain management, production planning, marketing decisions, cash flow management and financial planning (Kim et al. 2022).

In the context of portfolio selection, GP is an analytical approach devised to address financial decision-making problems where targets have been assigned to the attributes of a portfolio and where the decision maker is interested in minimizing the non-achievement of the corresponding goals. The main portfolio selection models utilize Weighted, Lexicographic and MinMax GP variants, but other variants include Fuzzy GP, Compromise Programming, Stochastic GP (Azmi 2013), Robust Optimization and Chance-Constrained Programming (Kim et al. 2022).

The Weighted Goal Programming (WGP) for portfolio selection lists the unwanted deviational variables weighted according to their importance and minimizes the sum of the unwanted weighted deviations. Besides penalizing excess risk and shortfalls in return relative to the respective targets, a WGP model can include attributes such as liquidity, cost of rebalancing and regional allocation (Azmi 2013).

$$\min \sum_{i=1}^{m} \alpha_i n_i + \beta_i p_i$$
$$\text{subject to: } \begin{cases} f_i(\mathbf{x}) + n_i - p_i = b_i, & i = 1, \ldots, m \\ \mathbf{x} \in C_S \\ \mathbf{x} \geq 0, \quad n_i, p_i \geq 0, & i = 1, \ldots, m \end{cases}$$

Fig 2: Theoretical Example of a Weighted Goal Program Model (Azmi 2013) .

Lexicographic Goal Programming (LGP) is also called Preemptive or Hierarchical Goal Programming. LGP achievement functions imply a non-compensatory structure of preferences. This means that a priority structure is established by ranking the goals in order of importance to the decision maker, although numerical weights can still represent the relative importance of the goals at the same priority level. Thus, LGP could deal with many priority levels and goal constraints can be included according to their importance of achievement in the model (Azmi 2013). It will surely be better visualized and understood through the following example.

Consider the following linear multi-objective optimization problem on the left (Arenales et al. 2007):

$$\begin{aligned} &\max\ f_1(\mathbf{x}) = \mathbf{c}^T\mathbf{x} \\ &\min\ f_2(\mathbf{x}) = \mathbf{d}^T\mathbf{x} \\ &\min\ f_3(\mathbf{x}) = \mathbf{e}^T\mathbf{x} \\ &\text{subject to: } \begin{cases} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \geq 0 \end{cases} \end{aligned}$$

- $f_1(\mathbf{x}) \geq \sigma_1$
- $f_2(\mathbf{x}) \leq \sigma_2$
- $f_3(\mathbf{x}) = \sigma_3$

Fig 3: Example of multi-objective optimization problem (Arenales et al. 2007).

Suppose that the decision maker is satisfied by any solution that attends the goals on the right of Fig. 3 (Arenales et al. 2007). This leads to the reformulation of the original problem in terms of finding a feasible solution to the system of equations, in which flexibility variables are introduced (Arenales et al. 2007):

$$\begin{aligned} &f_1(\mathbf{x}) + y_1 \geq \sigma_1 \\ &f_2(\mathbf{x}) - y_2 \leq \sigma_2 \\ &f_3(\mathbf{x}) + y_3^+ - y_3^- = \sigma_3 \\ &\text{subject to: } \begin{cases} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \geq 0, \quad y_1, y_2, y_3^+, y_3^- \geq 0 \end{cases} \end{aligned}$$

Fig 4: Multi-objective optimization problem reformulated with the usage of deviational variables (Arenales et al. 2007).

Under the perspective of WGP, the decision maker wants to minimize the deviational variables, so that they are null or almost null. Therefore, the following system must be solved (Arenales et al. 2007):

$$\min \phi(\mathbf{x},\mathbf{y}) = w_1 y_1 + w_2 y_2 + w_3(y_3^+ + y_3^-)$$
$$\text{subject to: } \begin{cases} f_1(\mathbf{x}) + y_1 \geq \sigma_1 \\ f_2(\mathbf{x}) - y_2 \leq \sigma_2 \\ f_3(\mathbf{x}) + y_3^+ - y_3^- = \sigma_3 \\ \mathbf{Ax} = \mathbf{b} \\ \mathbf{x} \geq 0, \quad y_1, y_2, y_3^+, y_3^- \geq 0 \end{cases}$$

Fig 5: Model for solving the example using Weighted Goal Programming (Arenales et al. 2007).

In the hierarchical approach, goals are taken in their priority order, one after the other. Let's say the goal priority is, in decreasing order: 1, 2, 3. So, we must first minimize the deviational variable y1, and after that y2 given y1 as fixed, and at last we minimize y3+ and y3− given the results obtained for y1 and y2. It is important to note that the set of constraints is adjusted according to each stage of the LGP process (Arenales et al. 2007).

$$\min \phi(\mathbf{x},\mathbf{y}) = y_1$$
$$\text{subject to: } \begin{cases} f_1(\mathbf{x}) + y_1 \geq \sigma_1 \\ \mathbf{Ax} = \mathbf{b} \\ \mathbf{x} \geq 0, \quad y_1 \geq 0 \end{cases}$$

Fig 6: First Step for solving the example using Lexicographic Goal Programming (Arenales et al. 2007).

$$\min \phi(\mathbf{x},\mathbf{y}) = y_2$$
$$\text{subject to: } \begin{cases} f_1(\mathbf{x}) + c_1 \geq \sigma_1 \\ f_2(\mathbf{x}) - y_2 \leq \sigma_2 \\ \mathbf{Ax} = \mathbf{b} \\ \mathbf{x} \geq 0, \quad y_2 \geq 0 \end{cases}$$

Fig 7: Second Step for solving the example using Lexicographic Goal Programming (Arenales et al. 2007).

$$\min \phi(\mathbf{x},\mathbf{y}) = y_3^+ + y_3^-$$
$$\text{subject to: } \begin{cases} f_1(\mathbf{x}) + c_1 \geq \sigma_1 \\ f_2(\mathbf{x}) - c_2 \leq \sigma_2 \\ f_3(\mathbf{x}) + y_3^+ - y_3^- = \sigma_3 \\ \mathbf{Ax} = \mathbf{b} \\ \mathbf{x} \geq 0, \quad y_3^+, y_3^- \geq 0 \end{cases}$$

Fig 8: Third and Last Step for solving the example using Lexicographic Goal Programming (Arenales et al. 2007).

OBJECTIVES

The present work aims at selecting funds for a portfolio, from a pool of funds traded openly at Stock Markets, using goal programming for achieving certain targets, mainly related to dividend yield and risk.

The type of investment chosen for the implementation of the Goal Programming model was Real State Investment Trusts - REIT (Fundo de Investimento Imobiliário - FII), which pools the capital of investors to invest in assets from the real state sector. Theses investments can range from purchase of properties, under construction or finished, for residential or commercial purposes, as well as acquisition of quotas from other trusts, Real State Credit Bonds (Letra de Crédito Imobiliário - LCI), Certificates of Real Estate Receivables (Certificado de Recebíveis Imobiliários - CRI), shares from real state companies, etc. A high percentage of profits must be distributed to quotaholders as dividends periodically, normally monthly (B3 2023a).

METHODOLOGY

A list of Brazilian REIT's, that are traded at the Brazilian Stock Exchange (B3 S.A. – Brasil, Bolsa, Balcão, B3 2023b), was obtained from the website Status Invest (Invest 2023), in which they were ranked by Equity Value (Patrimônio Líquido) in decreasing order. The 30 trusts on top of the list, for which data was fully and easily available at official websites, were selected for our study.

Therefore, a comprehensive Excel table, displayed in the Appendices section, was prepared for determination of ratios and indices used in the model, containing data like: Funds' categories, Price-to-Book Ratio P/B, History of Dividends by Quota, Mean Value (E), Median (M), Standard Deviation (σ), two Coefficients of Variations (σ/E and σ/M), Quota Prices, Conservative Dividend Yield and Adjusted Sharpe Ratio.

Their sectors and segments are standardized by ANBIMA (Associaçãoo Brasileira das Entidades dos Mercados Financeiro e de Capitais), the Brazilian Financial and Capital Markets Association (ANBIMA 2023). Based on that, the 30 selected funds comprise the following 7 categories:

1. Bonds and Securities (Títulos e Valores Mobiliários): 14 funds
2. Hybrid: 3 funds
3. Income - Hybrid: 3 funds
4. Income - Malls: 3 funds
5. Income - Corporate Slabs: 3 funds
6. Income - Logistics: 3 funds
7. Hybrid - Logistics: 1 fund

The variations of the model developed for this research, which is implemented using Julia Programming Language, with the aid of the Visual Studio Code IDE, intend mainly to mimic diferente profiles of investors, prioritizing return, or risk, or both, etc. For all of them, there's a set of common constraints, shown in the following figure:

$$
\text{subject to:} \begin{cases} \sum_{i=1}^{30} (y_i \cdot x_i) = 1 \\ \sum_{i=1}^{30} (y_i \cdot x_i \cdot CDY_{it}) + d1_t \geq Inf_t, \quad t = 1, ..., 6 \\ \sum_{i=1}^{30} y_i = 5 \\ y_i \cdot x_i \leq 0,50, \quad i = 1, ..., 30 \\ y_i \in \{0, 1\}, \quad x_i \geq 0 \quad (i = 1, ..., 30), \quad d1_t \geq 0 \quad (t = 1, ..., 6) \end{cases}
$$

Fig 9: Set of comoon constraints for all the variations of the model developed for this research.

Where:

• xi is a decision variable that represents the proportion of fund i in the portfolio (0 < xi < 1);

• yi is a binary decision variable that takes the value 1 when the fund is selected for the portfolio e 0 otherwise;

• t, which takes integer values from 1 to 6, represents the years from 2018 to 2023 respectively;

• CDYit represents the Conservative Dividend Yield for fund i at year t;

• d1t represents the deviational variable that does not allow this constraint to ever turn the system into unfeasible;

• Inft represents the Annual Inflation (IPCA) for year t;

The constraints have the following meanings and purposes:

1. The chosen funds must have proportions xi which have a sum of 1, so that they complete the portfolio;
2. The sum of the proportions of each selected fund multiplied by their respective Dividend Yield must surpass inflation at each year, whose data was taken from IBGE website (IBGE 2023), but the deviational variable allows for this target to be relaxed so that feasibility is held;
3. Exactly 5 funds must be selected;
4. No single fund can have a proportion which comprises more than half (0, 50 or 50%) of the whole portfolio;
5. Binary and non-negativity constraints.

RESULTS AND DISCUSSION

Presented at the Final Report.

CONCLUSIONS

Presented at the Final Report.

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